♣ 動差法：

設$X_1,\cdots,X_n$爲一組由p.d.f.$f(x|\boldsymbol{\theta})$所產生之隨機樣本, 其中$\boldsymbol{\theta} = (\theta_1, \cdots,\theta_k)$, k爲一正整數。令

$$\begin{aligned} m_1 &= \frac{1}{n}\sum_{i=1}^{n} X_i^1, \quad \mu_1 = E(X^1), \\ m_2 &= \frac{1}{n}\sum_{i=1}^{n} X_i^2, \quad \mu_2 = E(X^2), \\ &\vdots \\ m_k &= \frac{1}{n}\sum_{i=1}^{n} X_i^k, \quad \mu_k = E(X^k), \end{aligned}$$

此處$E(X^1)$即爲$E(X_1^1)$, 餘類推。對一組隨機樣本$X_1,\cdots,X_n$, 我們常以X代表其中的某一個隨機變數。由於母體的動差μ_j, $j \geq 1$, 爲參數$\theta_1,\cdots,\theta_k$之函數, 即$\mu_j = \mu_j(\boldsymbol{\theta}) = \mu_j(\theta_1,\cdots,\theta_k)$, 動差法就是由下述**動差方程式**

$$\begin{aligned} m_1 &= \mu_1(\theta_1,\cdots,\theta_k), \\ m_2 &= \mu_2(\theta_1,\cdots,\theta_k), \\ &\vdots \\ m_k &= \mu_k(\theta_1,\cdots,\theta_k), \end{aligned}$$

解出$\theta_1,\cdots,\theta_k$(將其表爲$m_1,\cdots,m_k$ 之函數), 以$\hat{\theta}_1,\cdots,\hat{\theta}_k$表此組解, 當做$\theta_1,\cdots,\theta_k$之估計量, 並稱做**動差法估計量**(method of moments estimator), 簡稱**動差估計量**。令$\hat{\boldsymbol{\theta}} = (\hat{\theta}_1,\cdots,\hat{\theta}_k)$。

動差估計量往往不唯一。爲了簡便, 通常我們採用由較低階動差所組成之估計量。底下列出常見分佈之期望值、變異數及動差估計量。

1. 伯努力分佈, $\mathcal{B}er(\theta)$

   $$E(X) = \theta, \qquad \mathrm{Var}(X) = \theta(1-\theta)$$

   動差估計量：

   $$\hat{\theta} = \overline{X}_n$$

2. 二項分佈, $\mathcal{B}(m,p)$, m,p皆未知

   $$E(X) = mp, \qquad \mathrm{Var}(X) = mp(1-p)$$

   動差估計量：

   $$\hat{m} = \frac{\overline{X}_n^2}{\overline{X}_n - \sum_{i=1}^{n}(X_i-\overline{X}_n)^2/n}, \qquad \hat{p} = \frac{\overline{X}_n}{\hat{m}}$$

3. 幾何分佈(自0開始), $\mathcal{G}e(\theta)$

$$E(X) = \frac{1-\theta}{\theta}, \qquad \mathrm{Var}(X) = \frac{1-\theta}{\theta^2}$$

動差估計量：

$$\hat{\theta} = \frac{1}{\overline{X}_n + 1}$$

4. 負二項分佈, $\mathcal{NB}(r,p)$, r,p皆未知

$$E(X) = \frac{r(1-p)}{p}, \qquad \mathrm{Var}(X) = \frac{r(1-p)}{p^2}$$

動差估計量：

$$\hat{r} = \frac{-\overline{X}_n^2}{\overline{X}_n - \sum_{i=1}^n (X_i - \overline{X}_n)^2/n}, \qquad \hat{p} = \frac{\overline{X}_n}{\sum_{i=1}^n (X_i - \overline{X}_n)^2/n}$$

5. 波松分佈, $\mathcal{P}(\lambda)$

$$E(X) = \lambda, \qquad \mathrm{Var}(X) = \lambda$$

動差估計量：

$$\hat{\lambda} = \overline{X}_n$$

6. 離散型均勻分佈, $\mathcal{D}\text{-}\mathcal{U}(1,\theta)$

$$E(X) = \frac{\theta+1}{2}, \qquad \mathrm{Var}(X) = \frac{\theta^2-1}{12}$$

動差估計量：

$$\hat{\theta} = 2\overline{X}_n - 1$$

7. 均勻分佈, $\mathcal{U}(\alpha,\beta)$, α,β皆未知

$$E(X) = \frac{\alpha+\beta}{2}, \qquad \mathrm{Var}(X) = \frac{(\beta-\alpha)^2}{12}$$

動差估計量：

$$\hat{\alpha} = \overline{X}_n - \sqrt{3}\Big(\sum_{i=1}^n (X_i - \overline{X}_n)^2/n\Big)^{\frac{1}{2}},$$

$$\hat{\beta} = \overline{X}_n + \sqrt{3}\Big(\sum_{i=1}^n (X_i - \overline{X}_n)^2/n\Big)^{\frac{1}{2}}$$

8. 指數分佈, $\mathcal{E}(\lambda)$

$$E(X) = \frac{1}{\lambda}, \qquad \mathrm{Var}(X) = \frac{1}{\lambda^2}$$

動差估計量：

$$\hat{\lambda} = \frac{1}{\overline{X}_n}$$

9. Gamma分佈, $\Gamma(\alpha,\beta)$, α,β皆未知

$$E(X)=\alpha\beta, \qquad \mathrm{Var}(X)=\alpha\beta^2$$

動差估計量：

$$\hat{\alpha}=\frac{\overline{X}_n^2}{\sum_{i=1}^n (X_i-\overline{X}_n)^2/n}, \qquad \hat{\beta}=\frac{\sum_{i=1}^n (X_i-\overline{X}_n)^2/n}{\overline{X}_n}$$

10. 常態分佈, $\mathcal{N}(\mu,\sigma^2)$, μ,σ^2皆未知

$$E(X)=\mu, \qquad \mathrm{Var}(X)=\sigma^2$$

動差估計量：

$$\hat{\mu}=\overline{X}_n, \qquad \hat{\sigma}^2=\frac{1}{n}\sum_{i=1}^n (X_i-\overline{X}_n)^2$$

11. Beta分佈, $\mathcal{B}e(r,s)$, r,s皆未知

$$E(X)=\frac{r}{r+s}, \qquad \mathrm{Var}(X)=\frac{rs}{(r+s)^2(r+s+1)}$$

動差估計量：

$$\hat{r}=\frac{m_1(m_1-m_2)}{-m_1^2+m_2}, \qquad \hat{s}=\frac{m_1-m_1^2-m_2+m_1m_2}{-m_1^2+m_2}$$

其中$m_1=\overline{X}_n$, $m_2=\frac{1}{n}\sum_{i=1}^n X_i^2$

12. 雙指數分佈, $\mathcal{D}$-$\mathcal{E}(\lambda)$

$$E(X)=0, \qquad \mathrm{Var}(X)=\frac{2}{\lambda^2}$$

動差估計量：

$$\hat{\lambda}=\left(\frac{2n}{\sum_{i=1}^n X_i^2}\right)^{\frac{1}{2}}$$

13. 卡方分佈, χ_m^2

$$E(X)=m, \qquad \mathrm{Var}(X)=m$$

動差估計量

$\hat{m}=\overline{X}_n$