1. (10 points) Please write down the five major assumptions in Regression Analysis.

2. (35 points) Consider a no-intercept model, i.e.

$$y = \beta_1 x + \beta_2 x^2 + \varepsilon,$$

where ε is a normal distribution with mean 0 and variance σ^2. Additionally we also assume that the errors are uncorrelated. Given n observations, $(y_i, x_i), i = 1, 2, \cdots, n, n > 2$. Let $Y = (y_1, \cdots, y_n)'$ and $X = \begin{pmatrix} x_1 & x_1^2 \\ \vdots & \vdots \\ x_n & x_n^2 \end{pmatrix}$.

a. Find the least-square estimator of $\beta = (\beta_1, \beta_2)'$, $\hat{\beta}$.

b. Show $\hat{\beta}$ is an unbiased estimator of β and find the covariance matrix of $\hat{\beta}$.

c. Give a geometrical interpretation of this least-square estimator.

d. Please write down SS_{Res} in terms of Y and X, and find the unbiased estimator of σ^2.

e. Find the MLE of β and compare this MLE with the least-square estimator.

3. (15 points) Assume the model is

$$Y = X\beta + \varepsilon,$$

where $Y = (y_1, \cdots, y_n)'$ is the vector of the observations; $X = \begin{pmatrix} x_{11} & x_{12} & \cdots & x_{1p} \\ x_{21} & x_{22} & \cdots & x_{2p} \\ \vdots & \vdots & \ddots & \vdots \\ x_{n1} & x_{n2} & \cdots & x_{np} \end{pmatrix}$ is the design matrix; $\beta = (\beta_1, \cdots, \beta_p)'$ is the vector of the unknown parameters, and $\varepsilon = (\varepsilon_1, \cdots, \varepsilon_n)'$ is a vector of errors with mean vector, $\mathbf{0}$, and covariance matrix, $\sigma^2 I_n$. Let $\hat{y}_{(i)}$ be the fitted value of the ith response based on all observations except the ith one, and define

$$e_{(i)} = y_i - \hat{y_{(i)}}$$

to be the ith prediction errors. Show that

$$e_{(i)} = \frac{e_i}{1 - h_{ii}},$$

where e_i is the original ith residual and h_{ii} is the ith diagonal elements of hat matrix.
(Hits: Let $X_{(i)}$ represent the original X matrix with the ith row x_i withheld. $h_{ii} = x_i'(X'X)^{-1}x_i$ and $[X_{(i)}'X_{(i)}]^{-1} = (X'X)^{-1} + \frac{(X'X)^{-1}x_i x_i'(X'X)^{-1}}{1-h_{ii}}$.)

4. (15 points) Consider the normal probability plot.

- **a.** How to construct this plot?
- **b.** What is the purpose of this plot?
- **c.** If the observations come from a heavy-tailed distribution, then please show the corresponding normal probability plot.

5. (15 points) Give 32 observations, $y_1, \cdots, y_{32}$, and assume the model is $y = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \varepsilon$. Please fill the following ANOVA table up.

Source Variation	Sum of Square	Degree of Freedom	Mean Square	F_0
Regression	972.9			
Residual				
Total	1237.54			

6. (10 points) Consider the following two models where $E(\varepsilon) = \mathbf{0}$ and $\mathrm{Var}(\varepsilon) = \sigma^2 I$:
Model A: $y = X_1\beta_1 + \varepsilon$
Model B: $y = X_1\beta_1 + X_2\beta_2 + \varepsilon$.
Show that $R_A^2 \leq R_B^2$.