1. Explain the following designs:

a. The completely randomized design (5 points)

b. The randomized complete block design (5 points)

c. The Graeco-Latin square design (5 points)

d. The two-factor factorial design (5 points)

2. Consider a randomized complete block design for a signal factor. Suppose there are a treatments and b blocks.

a. Write down its effects model. (5 points)

b. Show $SS_T = SS_{Treatment} + SS_{Block} + SS_E$ (10 points)

c. $E(MS_{Treatment}) = \sigma^2 + \frac{b\sum_{i=1}^{a}\tau_i^2}{a-1}$, where τ_i is the effect of the ith treatment, and σ^2 is the variance of the error term. (10 points)

3. Consider a balanced incomplete block designs (BIBD) with a treatments and b blocks. Assume that each block contains k treatments and each treatment occurs r times in the design.

a. What does "balance" mean? (5 points)

b. Construct a BIBD with $a = 4, b = 4, k = 3$ and $r = 3$. (10 points)

c. Find the number of times each pair of treatments appears in the same block. (10 points)

d. Verify that the BIBD with $a = 8, b = 16, r = 8$ and $k = 4$ does not exist. (10 points)

4. A mechanical engineer is studying the thrust force developed by a drill press. He suspects that the drilling speed and the feed rate of the material are the most important factors. He selects four feed rates and use a high and low drill speed chosen to represent the extreme operating conditions. He obtains the following results

Drill speed \ Freed Rate	0.015	0.030	0.045	0.060
125	2.70	2.45	2.60	2.75
	2.78	2.49	2.72	2.86
200	2.83	2.85	2.86	2.94
	2.86	2.80	2.87	2.88

a. Name an appropriate design for the above experiment. (5 points)

b. Write down the corresponding statistical model. (5 points)

c. Write down the ANOVA. (10 points)

5. Consider a randomized complete block design with a treatments and b blocks. Suppose the observation y_{ij} for the treatment i in block j is missing.

a. Find the estimate of the missing observation by minimizing SS_E. (10 points)

b. Assume we have the following data sets for RCBD. Use the iterative ap-

Treatments \ Blocks	1	2	3	4	5
1	73	68	74	71	67
2	73	67		72	70
3	75	68	78	73	68
4	73	71	75		69

proach to estimate y_{23} and y_{44} after two iterations, starting with $y_{23} = 70.5$. (10 points)

6. Consider a complete randomized design for a signal factor with 4 treatments. Suppose we have the following contrasts:

$$\begin{aligned} C_1 &= 3y_{1\cdot} - y_{2\cdot} - y_{3\cdot} - y_{4\cdot} \\ C_2 &= 2y_{2\cdot} - y_{3\cdot} - y_{4\cdot} \\ C_3 &= y_{3\cdot} - y_{4\cdot} \end{aligned}$$

a. Show these contrasts are the orthogonal contrasts. (5 points)

b. $SS_{Treatment} = SS_{C_1} + SS_{C_2} + SS_{C_3}$. (10 points)