1. (20 %) Explain the following designs:

a. The completely randomized design

b. The randomized complete block design

c. The balanced incomplete block design

d. The Latin square design

2. (20 %) Consider a two-factor factorial design. There are a levels of factor A and b levels of factor B. There are also n replicates.

a. Write down its effects model.

b. Write down the corresponding sum of squares, $SS_T, SS_A, SS_B, SS_{AB}$, and SS_E.

c. Show its analysis of variance table.

3. (15 %) Consider a completely randomized design for a signal factor with a treatments and n replicates. For the fixed effects model, the observation y_{ij} can be written as

$$y_{ij} = \mu + \tau_i + \varepsilon_{ij}, i = 1, \cdots, a, j = 1, \cdots, n,$$

where $\sum_{i=1}^{a} \tau_i = 0$ and ε_{ij} are i.i.d. $N(0, \sigma^2)$. Show

a. $SS_T = SS_{Treatment} + SS_E$

b. $E(MS_E) = \sigma^2$

c. $E(MS_{Treatment}) = \sigma^2 + \frac{n\sum_{i=1}^{a} \tau_i^2}{a-1}$

$* \ F_{0.05,2,6} = 5.41, F_{0.05,6,2} = 19.33, F_{0.025,2,6} = 7.26, F_{0.025,6,2} = 39.33, F_{0.05,3,6} = 4.76, F_{0.05,6,3} = 8.94, F_{0.025,7,8} = 4.53, F_{0.025,8,7} = 4.90, t_{0.05,15} = 1.753, t_{0.025,15} = 2.131$

4. (15 %) A new filtering device is installed in a chemical unit. Before its installation, a random sample yielded the following information about the percentage of impurity: $\bar{y}_1 = 12.5, S_1^2 = 101.17$, and $n_1 = 8$. After installation, a random sample yielded $\bar{y}_2 = 10.2, S_2^2 = 94.73$, and $n_2 = 9$.

a. Can you conclude that two variances are equal? Use $\alpha = 0.05$.

b. Has the filtering device reduced the percentage of impurity significantly? Use $\alpha = 0.05$.

5. (15 %) Three different washing solutions are being compared to study their effectiveness, and only three trials can be done on any day. Because days could represent a potential source of variability, the experimenter decides to use a "randomized complete block design". Observations are taken for four days. Analyze the data from this experiment (use $\alpha = 0.05$) and draw conclusions.

Solution \ Day	1	2	3	4
1	13	22	18	39
2	16	24	17	44
3	5	4	1	22

6. (15 %) Consider the following data from a two-factorial experiment. Analyze the data (using $\alpha = 0.05$) and draw the conclusion.

Factor 1 \ Factor 2	1	2	3	4
1	36	39	36	32
2	18	20	22	20
3	30	37	33	34