Option pricing under a jump-diffusion model with autoregressive jump structure

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Abstract

The jump-diffusion models have been used extensively for options pricing where jumps are generated by a Poisson process with serially independent jump sizes. Some important examples have been widely discussed in the financial literature, such as Merton (1976) and Kou (2002). However, in these models, the independence among successive jump sizes makes them restrictive in capturing the conditional hetero-skedasticity (serial correlation in stock volatility) as usually observed in the real markets. As a remedy, in this study we propose an extended version of Merton's jump-diffusion model such that its jump sizes are serially correlated with an AR(1) structure. We conduct a full mathematical analysis for the proposed model, and derive the main statistics of the return distribution. By using the fact that the stock price is log-normally distributed conditional on the number of jumps, the analytical formulas for European option prices are also derived.

Based on these analytical results, a number of numerical examples are provided to investigate the impact of AR correlation coefficient on return distributions, option prices, volatility smiles, and the weights of options and stocks in a delta-gamma neutral portfolio. In each respect, we take Merton's jump-diffusion model as a special benchmark case (no correlation in jumps), and look into the deviations caused by the nonzero correlation coefficient. It is observed that, when the successive jumps are positively correlated, the return distribution tends to be more widely spread and more heavy-tailed than the distribution under Merton's model. In addition, stronger correlation will lead to higher volatility in stock returns and in turn cause higher option prices. Moreover, as the correlation increases, the implied volatility smiles tend to show a more curved shape. Furthermore, the correlation also has significant influences on the weights of a delta-gamma neutral portfolio. We conclude that ignoring the serial correlation in jumps while the correlation is in presence may bring in significant errors.