1. Write down the purpose of each term: (20 points)

(1) Cook's D, $D_i(X'X, pMS_{Res}) = \frac{(\hat{\beta_{(i)}}-\hat{\beta})'(X'X)(\hat{\beta_{(i)}}-\hat{\beta})}{pMS_{Res}}$

(2) $DFBETAS_{j,i} = \frac{\hat{\beta}_j - \hat{\beta}_{j\,(i)}}{\sqrt{S_{(i)}^2 C_{jj}}}$

(3) $DFFITS_i = \frac{\hat{y_i} - \hat{y_{(i)}}}{\sqrt{S_{(i)}^2 h_{ii}}}$

(4) $COVRATIO_i = \frac{|(X_{(i)}'X_{(i)})^{-1}S_{(i)}^2|}{|(X'X)^{-1}MS_{Res}|}$.

2. Since $\hat{\beta_{(i)}} - \hat{\beta} = \frac{(X'X)^{-1}x_i e_i}{1-h_{ii}}$, show that $D_i = \frac{r_i^2}{p}(\frac{h_{ii}}{1-h_{ii}})$, where $r_i^2 = \frac{e_i^2}{MS_{Res}(1-h_{ii})}$ and p is the number of the parameters. (10 points)

3. Since $(X_{(i)}'X_{(i)})^{-1} = (X'X)^{-1} + \frac{(X'X)^{-1}x_i x_i'(X'X)^{-1}}{1-h_{ii}}$, show that $COVRATIO_i = [\frac{S_{(i)}^2}{MS_{Res}}]^p(\frac{1}{1-h_{ii}})$. (10 points)

4. Consider the simple linear regression model $y = \beta_0 + \beta_1 x + \varepsilon$, and $Var(\varepsilon) = \sigma^2(E(y))^\alpha, \alpha \neq 2$. Show that $y' = y^{1-\alpha/2}$ is a variance-stabilizing transformation. (10 points)

5. Assume the model is

$$Y = X\beta + \varepsilon,$$

where $E(\varepsilon) = \mathbf{0}, Var(\varepsilon) = \sigma^2 V$. Find the generalized least-squares estimator of β. (10 points)

6. Suppose we wish to fit the piecewise quadratic polynomial with a knot at $x = t$:

$$E(y) = S(x) = \beta_{00} + \beta_{01}x + \beta_{02}x^2 + \beta_{10}(x-t)_+^0 + \beta_{11}(x-t)_+^1 + \beta_{12}(x-t)_+^2.$$

(1) Show how to test the hypothesis that this quadratic spline model fits the data significantly better than an ordinary quadratic polynomial.(10 points)

(2) This quadratic spline polynomial model is not continuous at the knot t. How can the model be modified so that continuity at $x = t$ is obtained? (10 points)

(3) Show how the model can be modified so that both $E(y)$ and $dE(y)/dx$ are continuous at $x = t$. (10 points)

7. Write down the procedures of the following methods for variable selection: (1) Forward Selection, (2) Backward Elimination and (3) Stepwise regression. (30 points)