Homework 2

1. Define

$$f(x_1, x_2) = (x_1 - 2)^4 + (2x_2 - 5)^4.$$

Implement the gradient descent algorithm and the Newton-Raphson algorithm for finding the minimum of f, and then compare the performances of these two algorithm.

2. Consider a normal mixture model with equal variance and fixed weight, i.e.

$$\lambda N(\mu_0, \sigma^2) + (1 - \lambda) N(\mu_1, \sigma^2).$$

Let $\theta = (\lambda, \mu_0, \mu_1, \sigma^2)^T$ be the parameter vector.

(1) Write down the corresponding two steps in the EM algorithm.

(2) Implement the EM algorithm to find the MLE of θ.

(3) Give the priors of the parameters,

$$\begin{aligned} P(\lambda) &\sim Beta(a, b); \\ P(\mu_0) &\sim N(\alpha_0, \gamma_0^2); \\ P(\mu_1) &\sim N(\alpha_1, \gamma_1^2); \\ P(\sigma^2) &\sim inverse\ Chi-square(n_0, s_0). \end{aligned}$$

Find MLE of θ by the data augmentation algorithm, and the algorithm is to iterate the following two steps:

Step 1: Sample Z_i from $Ber(p_i)$, where $p_i = P(Z_i = 1 | Y_i, \theta)$.

Step 2: Update the parameters by their posterior means.

3. Let

$$f(x_1, x_2) = |(3 - 2x_1)x_1 - 2x_2 + 1|^{7/3} + |(3 - 2x_2)x_2 - x_1 + 1|^{7/3}.$$

Implement a Pattern Search Algorithm to find the minimum point of $f(x_1, x_2)$ with the initial point $(x_1^{(0)}, x_2^{(0)}) = (-0.9, -1.0)$, and the initial step size $\Delta_1 = 0.3$.