1. Let f_0 and f_1 be two probability density functions. The Kullback-Leibler information number is defined as

$$K(f_0, f_1) = E_0 \log \frac{f_0(X)}{f_1(X)} = \int \frac{f_0(x)}{f_1(x)} f_0(x) dx.$$

Show that $K(f_0, f_1) \geq 0$.

2. Let $X_1, \cdots, X_n$ be i.i.d. random variables with mean μ and finite variance σ^2. (a) Show $\sqrt{n}(\bar{X}_n - \mu) \xrightarrow{d} Z$, where $\bar{X}_n = \frac{1}{n}\sum_{i=1}^n X_i$ and $Z \sim N(0, \sigma^2)$. (b) Based on the result of (a), find the asymptotic distribution of $\bar{X}_n^2$.

3. Let $X_1, \cdots, X_n$ be i.i.d. uniform$(0, \theta)$, and $X_{(1)}, \cdots, X_{(n)}$ be the order statistics. Show that $X_{(1)}/X_{(n)}$ and $X_{(n)}$ are independent.

4. Let $X_1, \cdots, X_n$ be i.i.d. random variables with pdf

$$f(x|\theta) = \frac{1}{\sigma} \exp(-(x - \mu)/\sigma), x > \mu, \mu \in R, \sigma > 0.$$

Here $\theta = (\mu, \sigma)$. (a) Find a sufficient statistic for θ, (b) Find the method of moments estimator of θ, and (c) find the maximum likelihood estimator of θ.

5. Suppose that $X_1, \cdots, X_m$, the counts in cells $1, \cdots, m$, follow a multinomial distribution with a total count n and cell probabilities $p_1, \cdots, p_m$. Find the maximum likelihood estimators of $p_1, \cdots, p_m$.

6. Assume that $X_1, \cdots, X_n$ are i.i.d. exponential(λ), i.e. $f(x|\lambda) = \lambda \exp(-\lambda x), x > 0$. (a) Show the maximum likelihood estimator of λ is $\bar{X}_n = \sum_{i=1}^n X_i/n$. (b) Based on the MLE of λ, find an unbiased estimator of λ. (c) Show this unbiased estimator can not attach the Cramer-Rao Lower Bound.

7. Let $X_1, \cdots, X_n$ be i.i.d. exponential(λ). Find the best unbiased estimator of $P(X < x) = 1 - \exp(-\lambda x)$, where x is a fixed constant.

8. Let $X_1, \cdots, X_n$ be i.i.d $N(\mu, \sigma^2)$. Here we assume that both μ and σ^2 are unknown. (a) Show that $\bar{X}_n$ and $S^2 = \frac{1}{n-1}\sum_{i=1}^n (X_i - \bar{X}_n)^2$ are independent. (b)Find the best unbiased estimator of μ/σ, the signal to noise ratio.

9. Let $X_1, \cdots, X_n$ be i.i.d. uniform$(0, \theta)$. Find the best unbiased estimator of θ^k.