1. Find the general solutions to

$$\begin{pmatrix} 2 & 3 & 1 & -1 \\ 5 & 8 & 0 & 1 \\ 1 & 2 & -2 & 3 \end{pmatrix} \mathbf{x} = \begin{pmatrix} -1 \\ -13 \\ -11 \end{pmatrix}.$$

2. If $z = (G-F)y+(I-FA)w$, where G and F are generalized inverses of A, show that the solution $\tilde{x} = Gy+(GA-I)z$ to $Ax = y$ reduces to $\tilde{x} = Fy+(FA-I)w$.

3. Let $\mathbf{x} = (x_1, \cdots, x_n)^T$ be $N(\mu, V)$ and define $\mathbf{x} = (\mathbf{x}_1, \mathbf{x}_2)^T$, where $\mathbf{x}_1 = (x_1, \cdots, x_k)^T$ and $\mathbf{x}_2 = (x_{k+1}, \cdots, x_n)^T$. Then find the marginal distribution of $\mathbf{x}_1$ and the conditional distribution of $\mathbf{x}_1|\mathbf{x}_2$.

4. Derive the moment generating function of non-central χ^2, $\chi^2(n, \lambda)$, and then use this moment generating function to get the corresponding mean and variance.

5. When $\mathbf{x}$ is $N(\mu_1, I)$; $\mathbf{y}$ is $N(\mu_2, I)$ and the correlation matrix between $\mathbf{x}$ and $\mathbf{y}$ is R, what are the mean and variance of $\mathbf{x}^T A\mathbf{y}$?

6. When $\mathbf{x}$ is $N(\mu, V)$, two quadratic forms, $\mathbf{x}^T A\mathbf{x}$ and $\mathbf{x}^T B\mathbf{x}$, are considered here. Show that if $BVA = \mathbf{0}$, then $\mathbf{x}^T A\mathbf{x}$ and $\mathbf{x}^T B\mathbf{x}$ are independent.

7. When $\mathbf{y}$ has mean vector Xb and variance-covariance matrix V, prove that the covariance of the b.l.u.e.'s of $p'b$ and $q'b$ is $p'(X'V^{-1}X)^{-1}q$.

8. When $\mathbf{y}$ is $N(XB, \sigma^2 I)$, show that $SSE/(N-r)$ is an unbiased estimator of σ^2, and SSE is independent of SSR. Finally write down the distribution of

$$F(R) = \frac{SSR/r}{SSE/(N-r)}.$$

9. Consider the hypothesis $H : K'b = m$, and assume $\mathbf{y}$ is $N(Xb, \sigma^2 I)$. Find the least-squares estimator of b under this hypothesis.

10. Assume there is a sample of n observations $\mathbf{y} = (y_1, \cdots, y_n)'$, where $\mathbf{y}$ is $N(Xb, \sigma^2 I)$. Find the likelihood ratio test for $H : b = \mathbf{0}$