1. The observations are $X_i = Z_i + \varepsilon_i, i = 1, \cdots, n$, where Z_i are unobservable i.i.d. exponential random variables with mean $\theta > 0$ and the error terms ε_i are i.i.d. Bernoulli with parameter p, independent of the Z_i.

1. Find the method of moments estimates of θ and p. For what values of (θ, p) are these estimates consistent?
2. Show there is a two-dimensional sufficient statistic for (θ, p).
3. Find Fisher Information matrix, $I(\theta, p)$.

2. Let $X_1, \cdots, X_n$ be the sample from a mixture of gamma distributions:

$$f(x|\theta) = [(1-\theta)\exp(-x) + \theta x \exp(-x)], x > 0,$$

where $0 < \theta < 1$.

1. What is the estimate of θ given by the method of moments?
2. What is its asymptotic distribution?
3. Show how to improve this estimate by one iteration of Newton's method applied to the likelihood equation.

3. Each of n light bulbs with common exponential density for the time to failure is left on until it fails or until time T whichever occurs first. Thus the observations, $X_1, \cdots, X_n$, are i.i.d. with density $(1/\theta)\exp(-x/\theta)$ on $(0, T)$, and with $P(X_i = T) = \exp(-T/\theta)$.

1. Find the MLE of θ.
2. Find the asymptotic distribution of the MLE.

4. Consider the linear regression model with observations $Y_i = \alpha + \beta x_i + \varepsilon_i, i = 1, \cdots, n$, where x_i are known constants, α and β are unknown parameters, and ε_i are independent normal random errors with mean 0 and known variance, σ^2.

1. Find the Fisher Information matrix, $I(\alpha, \beta)$.
2. Using this Fisher Information matrix, find the Cramer-Rao lower bound for the variance of an unbiased estimate of β.

5. Let $X_1, \cdots, X_n$ be a sample from the Poisson distribution with density $f(x|\mu) = \exp(-\mu)\mu^x/x!$, and let $Y_1, \cdots, Y_n$ be an independent sample from the Poisson distribution with density $f(y|\theta)$, where $\mu > 0$ and $\theta > 0$ are unknown parameters.

1. Find the likelihood ratio test statistic for testing $H_0 : \mu = \theta^2$.
2. What is its asymptotic distribution?

6. N balls are distributed at random into $I \times J$ cells, where cell (i, j) has probability $p_{ij} > 0$, for $i = 1, \cdots, I$ and $j = 1, \cdots, J$, with $\sum_i \sum_j p_{ij} = 1$. Let n_{ij} represent the number of balls that fall in cell (i, j), $\sum_i \sum_j n_{ij} = N$.

1. Find the χ^2 test of the hypothesis $H : \sum_j p_{ij} = 1/I$, for $i = 1, \cdots, I$. How many degrees of freedom?
2. Find the χ^2 test of the hypothesis $H_0 : p_{ij}$ is independent of i, i.e. $p_{1j} = p_{2j} = \cdots = p_{Ij}$ for $j = 1, \cdots, J$. How many degrees of freedom?
3. χ^2 test of H_0 against $H - H_0$. How many degrees of freedom?